ANNEX 1

Proposal for Cornish-Fisher Methodology

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Cornish-Fisher Methodology

1. Retrieve prices form historical data: $P_1, P_2, \ldots, P_N$.
2. Compute daily log-returns
$$R_i = \log \frac{P_i}{P_{i-1}}, \qquad i = 1, \ldots, N.$$
3. Adjust for the sample mean, that is, consider
$$\widetilde{R}_i = R_i - \mu, \qquad \text{where } \mu = \frac{1}{N}\sum_{i=1}^{N} R_i.$$
4. Compute sample moments of $\widetilde{R}$ up to fourth order:
$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N} \widetilde{R}_i^2} \qquad \text{(standard deviation)},$$
$$\gamma = \frac{\frac{1}{N}\sum_{i=1}^{N} \widetilde{R}_i^3}{\sigma^3} \qquad \text{(skewness)},$$
$$\kappa = \frac{\frac{1}{N}\sum_{i=1}^{N} \widetilde{R}_i^4}{\sigma^4} - 3 \qquad \text{(excess kurtosis)}.$$
5. Compute the approximation of the α-quantile q_α of $\widetilde{R}$ by means of Cornish-Fisher expansion:
$$x = \Phi^{-1}(\alpha), \qquad \Phi = \text{ standard normal distribution probability function},$$
$$\omega = x + \frac{\gamma}{6}(x^2 - 1) + \frac{\kappa}{24}x(x^2 - 3) - \frac{\gamma^2}{36}x(2x^2 - 5),$$
$$q_\alpha = \sigma\omega.$$
6. The VaR for 1 day with confidence level α is $VaR_{\alpha,1} = -q_\alpha$.
7. Transform VaR to equivalent daily volatility as follows: assuming that $\widetilde{R}_i$, $i = 1, \ldots, N$ is a sample of a log-normal distribution with mean 1 and standard deviation v, then v and $VaR_{\alpha,1}$

would be related by:

$$VaR_{\alpha,1} = \frac{v^2}{2} - v\Phi^{-1}(\alpha).$$

Therefore, v is the positive solution of

$$\frac{v^2}{2} + 1.96v - VaR_{\alpha,1} = 0,$$

that is

$$v = -1.96 + \sqrt{1.96^2 + 2VaR_{\alpha,1}}.$$

8. Finally, annualized equivalent volatility is

$$\text{Vol } = v\sqrt{\sharp \text{ trading days in one year}}.$$

Comments

Estimators

The estimators for the moments γ, and κ presented in the previous section are the classical estimators, and it is well known that they are biased. In order to reduce bias the following estimators are often used:

$$\gamma = \frac{\frac{N}{(N-1)(N-2)}\sum_{i=1}^{N}\widetilde{R}_i^3}{\sigma^3},$$
$$\kappa = \frac{\frac{N(N+1)}{(N-1)(N-2)(N-3)}\sum_{i=1}^{N}\widetilde{R}_i^4}{\sigma^4} - 3\frac{(N-1)^2}{(N-2)(N-3)},$$

Returns expected value

In order to impose $\widetilde{R}$ to have expected value equal to the risk free rate we should have defined $\widetilde{R}_i = R_i - \mu + r$, where r is the risk free rate. However, the effect of adding r would be undone later when discounting the VaR.

Methodology drawbacks

Cornish-Fisher methodology yields a very poor approximation of a distribution's quantile when the distribution is "far" form the normal distribution, for example for large skewness or kurtosis. The following example shows how just a few isolated extreme returns have a big effect on skewness and kurtosis, making Cornish-Fisher methodology incurring in large errors.

- Fund's Name: Liberbank Ahorro, FI.
- ISIN: ES0111037034.
- Data time span: 11/24/2010 - 11/24/2015.
- Data source: Bloomberg.

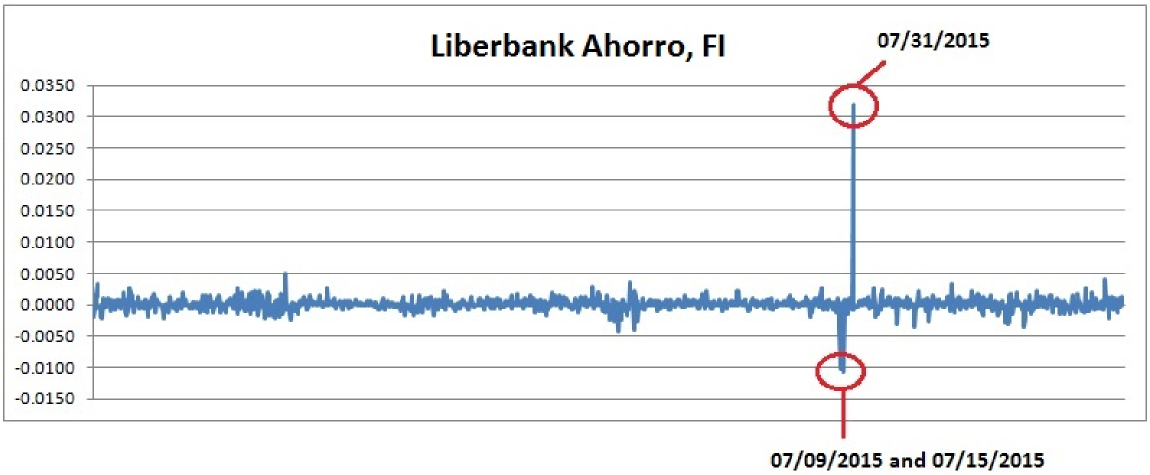

Figure 1: Liberbank Ahorro, FI, daily log-returns

The following table shows the difference between the sample quantile and the Cornish-Fisher approximation, and the difference between the annualized equivalent volatility computed from Cornish-Fisher approximation and from UCITS methodology. The observed large errors are due to the large skewness and kurtosis of this sample distribution.

α	2.5%
Skewness	9.34
Kurtosis	224.59
Sample quantile	-0.00195
Cornish-Fisher quantile	-0.00020
Cornish-Fisher annualized volatility	0.05%
Annualized volatility within UCITS framework	1.18%